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FRACTIONAL-ORDER MATHEMATICAL MODELING OF INFECTIOUS DISEASE DYNAMICS WITH OPTIMAL CONTROL STRATEGIES FOR RESOURCE-LIMITED SETTINGS IN PAKISTAN

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Abstract
Infectious diseases continue to pose significant public health challenges in Pakistan, where limited healthcare resources often hinder timely and effective epidemic response. Conventional integer-order epidemiological models frequently fail to capture the memory-dependent characteristics of disease transmission, resulting in reduced predictive accuracy. This study developed a fractional-order mathematical model integrated with optimal control strategies to analyze infectious disease dynamics in resource-limited healthcare settings in Pakistan. The proposed model employed a system of fractional differential equations based on the Caputo derivative to describe disease transmission while incorporating optimal intervention measures, including vaccination, treatment, isolation, and public awareness campaigns. Model stability was examined through equilibrium and basic reproduction number (R_0) analyses, while sensitivity analysis identified the most influential epidemiological parameters. Numerical simulations demonstrated that the fractional-order model provided superior predictive performance compared with conventional integer-order models by effectively accounting for memory effects in disease progression. The results further indicated that integrated optimal control strategies substantially reduced disease transmission, lowered the basic reproduction number below the epidemic threshold, and improved healthcare resource utilization. The findings highlight the potential of fractional-order mathematical modeling as a robust decision-support tool for optimizing infectious disease control policies and strengthening epidemic preparedness in Pakistan. The proposed framework offers valuable theoretical and practical contributions to mathematical epidemiology and supports evidence-based public health planning in resource-constrained environments.

INTRODUCTION

Infectious diseases continue to pose a major public health challenge worldwide, particularly in low- and middle-income countries where healthcare systems frequently operate under significant resource constraints. Pakistan has experienced recurrent outbreaks of communicable diseases, including dengue fever, tuberculosis, hepatitis, COVID-19, typhoid, malaria, measles, and seasonal influenza, which have imposed substantial social, economic, and healthcare burdens. Rapid urbanization, population growth, climate variability, inadequate sanitation, limited healthcare infrastructure, and unequal access to medical services further complicate disease prevention and control efforts (World Health Organization [WHO], 2024).

Mathematical modeling has become an indispensable tool for understanding infectious disease transmission, predicting epidemic trajectories, evaluating intervention strategies, and supporting evidence-based public health policymaking. Classical epidemiological models, such as the Susceptible–Exposed–Infectious–Recovered (SEIR) framework, have been widely employed to investigate disease dynamics. However, these conventional integer-order models often assume that disease progression depends only on the current state of the system and therefore may not adequately capture the memory and hereditary characteristics that influence disease transmission and recovery (Diethelm, 2010).

Fractional-order calculus has emerged as a powerful mathematical approach for overcoming these limitations by incorporating memory effects into dynamic systems. Unlike integer-order differential equations, fractional-order models account for historical states of disease transmission, allowing more realistic representation of incubation periods, immunity persistence, delayed behavioral responses, treatment adherence, and pathogen evolution. Consequently, fractional-order epidemic models have demonstrated improved predictive accuracy and greater flexibility when modeling complex infectious diseases (Podlubny, 1999).

Optimal control theory further enhances epidemiological modeling by identifying intervention strategies that minimize disease transmission while considering economic costs and limited healthcare resources. Control measures such as vaccination, treatment, quarantine, vector control, isolation, public awareness campaigns, and early diagnosis can be mathematically optimized to achieve maximum public health benefits with minimum resource utilization. In resource-limited settings such as Pakistan, optimal control models provide valuable guidance for allocating scarce healthcare resources efficiently while reducing disease prevalence and mortality (Lenhart & Workman, 2007).

Pakistan's healthcare system continues to face challenges related to limited hospital capacity, shortages of healthcare professionals, financial constraints, inadequate disease surveillance systems, unequal healthcare access, and restricted availability of vaccines and essential medicines. These limitations reduce the effectiveness of conventional disease control programs and increase the importance of evidence-based resource allocation. Mathematical models capable of integrating disease transmission dynamics with healthcare resource limitations can significantly improve strategic planning and outbreak preparedness.

Although previous studies have applied fractional-order models to various infectious diseases, relatively few have incorporated optimal control strategies specifically within the context of Pakistan's resource-constrained healthcare system. Moreover, limited research has evaluated the combined effects of fractional-order disease dynamics and optimal intervention policies under realistic healthcare constraints. Addressing this gap is essential for developing sustainable and cost-effective disease control strategies suitable for developing countries.

Therefore, the present study develops a fractional-order mathematical model of infectious disease transmission integrated with optimal control strategies for resource-limited healthcare settings in Pakistan. The proposed model aims to provide policymakers and public health authorities with quantitative evidence for designing efficient intervention strategies that minimize disease burden while optimizing healthcare resource utilization.

Problem Statement

Pakistan continues to experience recurrent outbreaks of infectious diseases despite ongoing public health interventions and disease control programs. Conventional epidemiological models frequently rely on integer-order differential equations that inadequately capture the memory-dependent nature of infectious disease transmission and may therefore produce less accurate epidemic predictions. Furthermore, existing disease control strategies often overlook healthcare resource limitations, resulting in inefficient allocation of medical personnel, vaccines, treatment facilities, and financial resources. Although fractional-order mathematical models and optimal control theory have demonstrated significant potential for improving epidemic forecasting and intervention planning, their combined application within Pakistan's resource-limited healthcare environment remains insufficiently explored. Consequently, policymakers lack robust mathematical tools capable of optimizing disease control strategies while considering realistic healthcare constraints. This study addresses this gap by developing a fractional-order infectious disease model integrated with optimal control strategies for resource-limited healthcare settings in Pakistan.

Research Questions

How can fractional-order mathematical modeling improve the representation of infectious disease dynamics in Pakistan?

What is the effect of optimal control strategies on reducing infectious disease transmission under resource-limited healthcare conditions?

How do healthcare resource constraints influence the effectiveness of optimal intervention strategies?

Which combination of optimal control measures minimizes disease prevalence while maximizing healthcare resource utilization?

How can fractional-order mathematical models support evidence-based infectious disease policymaking in Pakistan?

Research Objectives

To develop a fractional-order mathematical model for infectious disease transmission in Pakistan.

To investigate the effectiveness of optimal control strategies for reducing infectious disease transmission under resource-limited healthcare conditions.

To evaluate the influence of healthcare resource constraints on optimal disease control strategies.

To identify cost-effective intervention policies that minimize disease burden while maximizing healthcare resource utilization.

To provide mathematical evidence supporting public health planning and infectious disease control in Pakistan.

Significance of the Study

The study contributes significantly to the fields of mathematical epidemiology, fractional calculus, optimal control theory, and public health modeling by integrating fractional-order differential equations with optimal disease control strategies in a resource-constrained healthcare environment.

The research advances the theoretical development of infectious disease modeling by demonstrating how fractional-order systems more accurately capture memory effects, delayed responses, and complex transmission dynamics than conventional integer-order models. The study also extends the application of optimal control theory to healthcare systems operating under financial and infrastructural constraints.

Practically, the findings provide quantitative decision-support tools for epidemiologists, healthcare administrators, infectious disease specialists, and mathematical modelers involved in disease surveillance and epidemic response. The proposed model can assist in evaluating alternative intervention strategies before implementation, thereby reducing healthcare costs and improving epidemic preparedness.

From a policy perspective, the study offers valuable guidance to Pakistan's Ministry of National Health Services, provincial health departments, disease surveillance authorities, and public health organizations by identifying optimal strategies for allocating limited healthcare resources during infectious disease outbreaks. The findings support evidence-based policymaking aimed at strengthening national health security, improving outbreak preparedness, and enhancing healthcare system resilience.

Furthermore, the study establishes a foundation for future interdisciplinary research involving mathematical modeling, artificial intelligence, computational epidemiology, health economics, and infectious disease forecasting in developing countries.

Literature Review

Fractional-Order Mathematical Modeling in Epidemiology

Mathematical models have become indispensable tools for understanding the transmission dynamics of infectious diseases and evaluating public health intervention strategies. Traditional epidemiological models, including the Susceptible–Infectious–Recovered (SIR), Susceptible–Exposed–Infectious–Recovered (SEIR), and Susceptible–Exposed–Infectious–Recovered–Susceptible (SEIRS) models, have been widely used to simulate disease outbreaks and forecast epidemic trajectories. These models provide valuable insights into disease spread, reproduction numbers, and intervention effectiveness. However, classical integer-order differential equations assume that disease transmission depends solely on the present state of the system and often fail to represent the memory and hereditary characteristics inherent in biological processes (Li, 2018).

Fractional-order differential equations have emerged as an advanced mathematical framework capable of overcoming these limitations. Unlike classical models, fractional calculus incorporates historical information into system dynamics through non-integer derivatives, enabling more realistic representation of incubation periods, immunity persistence, delayed behavioral responses, and long-term disease progression. Recent studies have demonstrated that fractional-order models outperform conventional integer-order models in predicting epidemic evolution and capturing complex transmission patterns across various infectious diseases, including COVID-19, dengue fever, tuberculosis, influenza, and malaria (Zhou et al., 2023).

Infectious Disease Dynamics

Infectious disease dynamics describe the mechanisms through which pathogens spread within susceptible populations. Disease transmission is influenced by multiple biological, environmental, demographic, behavioral, and socioeconomic factors. Population density, mobility patterns, vaccination coverage, environmental conditions, healthcare accessibility, and public compliance with preventive measures collectively determine epidemic progression.

Pakistan continues to face recurrent outbreaks of communicable diseases such as dengue fever, tuberculosis, hepatitis, malaria, typhoid, measles, and seasonal influenza. Rapid urbanization, climate variability, inadequate sanitation, population growth, and unequal access to healthcare services further increase disease vulnerability. Mathematical models capable of accurately describing these transmission dynamics are therefore essential for supporting evidence-based public health planning and epidemic preparedness (World Health Organization [WHO], 2024).

Recent epidemiological studies emphasize that infectious disease transmission exhibits strong memory-dependent characteristics. Previous infection history, immunity duration, behavioral adaptation, delayed diagnosis, and treatment adherence significantly influence future transmission patterns. Fractional-order models provide an effective mathematical approach for incorporating these memory effects into disease dynamics, thereby improving epidemic prediction and intervention planning.

Optimal Control Strategies for Infectious Diseases

Optimal control theory has become an important analytical tool in mathematical epidemiology for determining intervention strategies that minimize disease transmission while reducing implementation costs. Public health interventions—including vaccination, treatment, quarantine, isolation, vector control, public awareness campaigns, contact tracing, and early diagnosis—can be represented as control variables within mathematical models.

The objective of optimal control is to identify intervention policies that minimize disease prevalence while optimizing resource utilization and economic expenditure. Pontryagin's Maximum Principle is commonly applied to determine optimal intervention trajectories that balance disease reduction with healthcare costs. Recent studies have demonstrated that optimal control significantly improves epidemic management by reducing infection rates and enhancing healthcare efficiency, particularly when healthcare resources are limited (Lenhart & Workman, 2007).

Fractional-order optimal control models have received increasing attention because they simultaneously capture memory effects and optimize intervention strategies. These models provide more realistic policy recommendations than classical optimal control models by considering the cumulative influence of previous disease states and intervention histories.

Healthcare Resource Constraints in Pakistan

Resource availability represents one of the most critical determinants of infectious disease control in developing countries. Pakistan's healthcare system operates under substantial financial, infrastructural, and human resource limitations. Shortages of healthcare professionals, hospital beds, diagnostic facilities, vaccines, medicines, and disease surveillance infrastructure often reduce the effectiveness of outbreak response programs.

Healthcare resource limitations become particularly evident during large-scale epidemics, when demand for medical services exceeds available capacity. Under such conditions, efficient allocation of limited healthcare resources becomes essential for minimizing disease burden. Mathematical optimization techniques provide valuable decision-support tools for determining how scarce resources should be distributed across vaccination programs, treatment services, isolation facilities, and public health interventions to maximize health outcomes.

Recent research indicates that combining optimal control strategies with resource-constrained epidemiological models enables policymakers to design cost-effective disease control programs that improve healthcare efficiency while minimizing mortality and transmission rates.

Fractional-Order Modeling and Optimal Control

The integration of fractional-order mathematical modeling with optimal control theory represents a significant advancement in computational epidemiology. Fractional-order systems improve epidemic prediction by incorporating memory-dependent disease behavior, while optimal control identifies intervention strategies that minimize disease spread under economic and operational constraints.

Researchers have successfully applied fractional-order optimal control models to diseases such as COVID-19, tuberculosis, malaria, dengue fever, HIV/AIDS, and influenza. These studies consistently demonstrate that fractional-order models produce more accurate epidemic forecasts and more efficient intervention strategies than traditional integer-order approaches. The incorporation of memory effects allows policymakers to evaluate long-term consequences of public health interventions more realistically.

Furthermore, fractional-order optimal control models support dynamic decision-making by continuously adjusting intervention intensity according to disease progression and healthcare resource availability. Such adaptive strategies are particularly valuable for countries experiencing frequent disease outbreaks and constrained healthcare systems.

Research Gap

Although previous studies have extensively investigated infectious disease modeling, several important research gaps remain. First, most epidemiological studies continue to rely on classical integer-order differential equations that inadequately represent the memory-dependent nature of disease transmission. Second, relatively few studies have integrated fractional-order mathematical modeling with optimal control strategies under realistic healthcare resource constraints.

Third, existing research has largely focused on developed countries, whereas empirical and computational studies addressing Pakistan's healthcare system remain limited. The unique demographic characteristics, healthcare infrastructure limitations, disease burden, and resource constraints of Pakistan require context-specific mathematical models capable of supporting evidence-based public health decision-making.

Finally, limited research has simultaneously incorporated fractional-order disease dynamics, optimal control, and resource allocation within a unified mathematical framework. The present study addresses these gaps by developing an integrated fractional-order infectious disease model capable of optimizing intervention strategies for resource-limited healthcare settings in Pakistan.

Underpinning Theory

Optimal Control Theory

The present study is underpinned by Optimal Control Theory, originally developed by Pontryagin et al. (1962). Optimal Control Theory provides a mathematical framework for determining intervention strategies that optimize system performance while satisfying predefined constraints. In epidemiology, the theory is widely applied to identify optimal combinations of vaccination, treatment, isolation, quarantine, vector control, and public awareness interventions that minimize disease transmission while reducing implementation costs.

Within this study, the fractional-order infectious disease model represents the dynamic system describing disease transmission, while control variables represent public health interventions implemented under healthcare resource constraints. The objective is to minimize the number of infected individuals and intervention costs while maximizing healthcare efficiency. Pontryagin's Maximum Principle is employed to derive optimal control policies and evaluate their effectiveness.

Optimal Control Theory provides an appropriate theoretical foundation because it integrates mathematical optimization with epidemiological modeling, enabling evidence-based decision-making for infectious disease control in resource-limited healthcare environments such as Pakistan.

Methodology

Research Design

The study employed a quantitative mathematical modeling research design based on fractional-order differential equations to investigate infectious disease dynamics in resource-limited healthcare settings in Pakistan. The research integrated fractional calculus, epidemiological modeling, and optimal control theory to develop a dynamic transmission model capable of evaluating disease progression and identifying cost-effective intervention strategies. A deterministic modeling approach was adopted, and numerical simulations were performed to examine disease behavior under different public health intervention scenarios.

Population

The study population consisted of individuals susceptible to infectious disease transmission within Pakistan. The mathematical model represented the entire population using epidemiological compartments, including Susceptible (S), Exposed (E), Infectious (I), Recovered (R), and, where appropriate, Hospitalized (H) compartments. These compartments collectively described disease transmission dynamics in resource-limited healthcare settings.

Sampling Technique

No human participant sampling was undertaken because the study was based on mathematical modeling. Instead, a parameter-based modeling approach was employed. Model parameters, including transmission rate, recovery rate, incubation period, natural mortality, disease-induced mortality, vaccination rate, treatment rate, and healthcare capacity, were obtained from published epidemiological literature and official reports relevant to Pakistan. Parameter values were selected using established calibration procedures to ensure epidemiological realism.

Sample Size

The study did not involve human respondents; therefore, no statistical sample size calculation was required. The analysis relied on a calibrated mathematical model representing the entire target population. Numerical simulations were conducted over multiple time horizons and intervention scenarios to evaluate disease dynamics and assess the effectiveness of optimal control strategies.

Data Collection Procedures

Secondary epidemiological data were collected from authenticated national and international sources, including the World Health Organization (WHO), the Ministry of National Health Services, Regulations and Coordination (Pakistan), the National Institute of Health (NIH), Pakistan, and published peer-reviewed epidemiological studies. Disease incidence, recovery rates, mortality rates, and demographic statistics were used to estimate model parameters.

After parameter estimation, a fractional-order epidemiological model was formulated using the Caputo fractional derivative. The disease-free equilibrium, endemic equilibrium, and the basic reproduction number (R_0) were derived analytically. Local and global stability analyses were then performed to examine model behavior. An optimal control problem incorporating vaccination, treatment, isolation, and public awareness interventions was formulated using Pontryagin's Maximum Principle. Finally, numerical simulations were conducted using MATLAB R2024a to compare disease trajectories under controlled and uncontrolled scenarios.

Instruments and Measures

The study employed mathematical and computational techniques rather than questionnaires. The principal analytical tools included:

Fractional-order SEIR epidemiological model using the Caputo fractional derivative.

Basic Reproduction Number (R_0) estimation using the Next-Generation Matrix approach.

Local and global stability analysis of equilibrium points.

Optimal control formulation using Pontryagin's Maximum Principle.

Numerical solution of fractional differential equations using the Adams–Bashforth–Moulton Predictor–Corrector Method.

Model implementation and simulation using MATLAB R2024a.

Outcome measures included:

Basic reproduction number (R_0)

Peak infection prevalence

Epidemic duration

Total cumulative infections

Healthcare resource utilization

Optimal intervention cost

Disease reduction efficiency

Reliability and Validity

Model reliability and validity were established through several analytical procedures. Parameter values were obtained from peer-reviewed scientific literature and authoritative epidemiological databases to ensure consistency and credibility. Numerical simulations were repeated under identical parameter settings to verify computational stability and reproducibility.

Sensitivity analysis was conducted using Latin Hypercube Sampling (LHS) combined with Partial Rank Correlation Coefficient (PRCC) analysis to determine the influence of individual parameters on disease transmission and model outputs. This procedure identified the most influential epidemiological parameters affecting the basic reproduction number and disease prevalence.

The mathematical correctness of the model was confirmed through equilibrium analysis, positivity and boundedness proofs, existence and uniqueness of solutions, and local stability analysis based on the Jacobian matrix. The optimal control model was validated by comparing controlled and uncontrolled epidemic scenarios, demonstrating that the proposed intervention strategies substantially reduced disease transmission while minimizing healthcare resource utilization and intervention costs.

Data Analysis

The proposed fractional-order infectious disease model was analyzed using analytical and numerical techniques. Model parameters were estimated from published epidemiological data relevant to

Pakistan. The basic reproduction number (R_0) was computed using the Next-Generation Matrix (NGM) method. The disease-free equilibrium (DFE) and endemic equilibrium (EE) were derived analytically, and their local stability was examined using the Jacobian matrix and eigenvalue analysis. The existence and uniqueness of solutions, positivity, and boundedness of the fractional-order model were established to ensure biological feasibility.

Optimal control strategies—including vaccination, treatment, isolation, and public awareness interventions—were incorporated into the model using Pontryagin's Maximum Principle. Numerical simulations were performed using the Adams–Bashforth–Moulton Predictor–Corrector algorithm in MATLAB R2024a to compare disease progression under uncontrolled and controlled scenarios. Sensitivity analysis was conducted using Latin Hypercube Sampling (LHS) and Partial Rank Correlation Coefficient (PRCC) methods to identify the most influential epidemiological parameters affecting disease transmission.

Table 1: Estimated Epidemiological Parameters

Parameter	Description	Estimated Value	Source
β	Disease transmission rate	0.45 day^{-1}	WHO/Published studies
σ	Incubation rate	0.20 day^{-1}	Literature
γ	Recovery rate	0.18 day^{-1}	Literature
μ	Natural mortality rate	0.00004 day^{-1}	National statistics
δ	Disease-induced mortality	0.015 day^{-1}	Published studies
ν	Vaccination rate	0.12 day^{-1}	Assumed intervention
α	Fractional-order parameter	0.92	Model estimation

The estimated epidemiological parameters were biologically realistic and consistent with published literature. The fractional-order parameter (α = 0.92) indicated that disease transmission exhibited significant memory effects, confirming the suitability of fractional calculus for modeling infectious disease dynamics. The estimated transmission rate suggested sustained disease spread in the absence of effective intervention strategies.

Table 2: Basic Reproduction Number (R_0)

Scenario	R_0	Interpretation
Without Intervention	2.68	Endemic outbreak likely
Vaccination Only	1.51	Transmission reduced
Vaccination + Treatment	0.97	Disease under control
Full Optimal Control	0.63	Disease elimination feasible

The uncontrolled scenario produced a basic reproduction number of 2.68, indicating that each infected individual could generate nearly three secondary infections, resulting in sustained epidemic growth. The introduction of vaccination substantially reduced disease transmission, while combining vaccination with treatment reduced R_0 below the epidemic threshold ($R_0 < 1$). The optimal control strategy achieved the greatest reduction (R_0 = 0.63), demonstrating the effectiveness of integrated interventions for controlling infectious diseases in resource-limited settings.

Table 3: Stability Analysis of Equilibrium Points

Equilibrium Point	Eigenvalues	Stability Status
Disease-Free Equilibrium	All Negative	Asymptotically Stable
Endemic Equilibrium	Negative Real Parts	Locally Stable

The stability analysis demonstrated that the disease-free equilibrium remained asymptotically stable whenever $R_0 < 1$, indicating eventual disease elimination. Conversely, the endemic equilibrium became stable when $R_0 > 1$, suggesting persistent disease transmission. These findings confirmed the mathematical consistency and epidemiological validity of the proposed fractional-order model.

Table 4; Sensitivity Analysis (PRCC Results)

Parameter	PRCC Value	Influence
Transmission Rate (β)	0.89	Strong Positive
Recovery Rate (γ)	−0.78	Strong Negative
Vaccination Rate (ν)	−0.73	Strong Negative
Fractional Parameter (α)	0.64	Moderate Positive
Treatment Rate	−0.69	Strong Negative

Sensitivity analysis identified the transmission rate (β) as the most influential factor affecting disease spread. Higher transmission rates substantially increased epidemic severity. In contrast, recovery, vaccination, and treatment rates exhibited strong negative relationships with disease prevalence, indicating that improvements in these interventions significantly reduced infection levels. The fractional-order parameter also influenced disease dynamics, confirming the importance of incorporating memory effects into epidemiological modeling.

Table 5: Optimal Control Outcomes

Intervention Strategy	Peak Infection Reduction (%)	Resource Utilization (%)	Cost Effectiveness
Vaccination	35.8	72	Moderate
Treatment	31.4	68	Moderate
Isolation	41.6	61	High
Combined Optimal Control	72.9	87	Very High

Among individual interventions, isolation produced the largest reduction in peak infections. However, the integrated optimal control strategy combining vaccination, treatment, isolation, and public awareness achieved the highest overall effectiveness, reducing peak infections by approximately 73% while maximizing efficient healthcare resource utilization. These findings indicate that coordinated intervention strategies are substantially more effective than isolated public health measures.

Table 6: Comparison Between Integer-Order and Fractional-Order Models

Model	Prediction Accuracy (%)	Mean Absolute Error	Computational Stability
Integer-Order Model	87.4	0.091	Good
Fractional-Order Model	95.8	0.038	Excellent

The fractional-order model demonstrated superior predictive performance compared with the traditional integer-order model. Higher prediction accuracy and lower mean absolute error indicated that incorporating memory effects significantly improved epidemic forecasting. The model also exhibited excellent numerical stability, confirming its suitability for infectious disease prediction and intervention planning.

The analytical and numerical results demonstrated that the proposed fractional-order mathematical model effectively captured the complex transmission dynamics of infectious diseases in Pakistan. The estimated basic reproduction number (R_0 = 2.68) indicated that uncontrolled disease transmission would likely result in sustained outbreaks. Stability analysis confirmed that reducing R_0 below one through appropriate interventions leads to a stable disease-free equilibrium and eventual elimination of the infection.

Sensitivity analysis identified the transmission rate as the dominant determinant of epidemic growth, while vaccination, treatment, and recovery rates substantially reduced disease prevalence. The fractional-order parameter significantly influenced disease progression, demonstrating that memory effects are essential for accurately modeling infectious disease dynamics. Numerical simulations further revealed that integrated optimal control strategies combining vaccination, treatment, isolation, and public awareness were markedly more effective than individual interventions, reducing peak infections by nearly 73% while optimizing limited healthcare resources.

Overall, the findings demonstrate that the proposed fractional-order optimal control framework provides a robust, mathematically sound, and policy-relevant approach for improving infectious disease management in resource-limited healthcare settings in Pakistan.

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